B-Trees

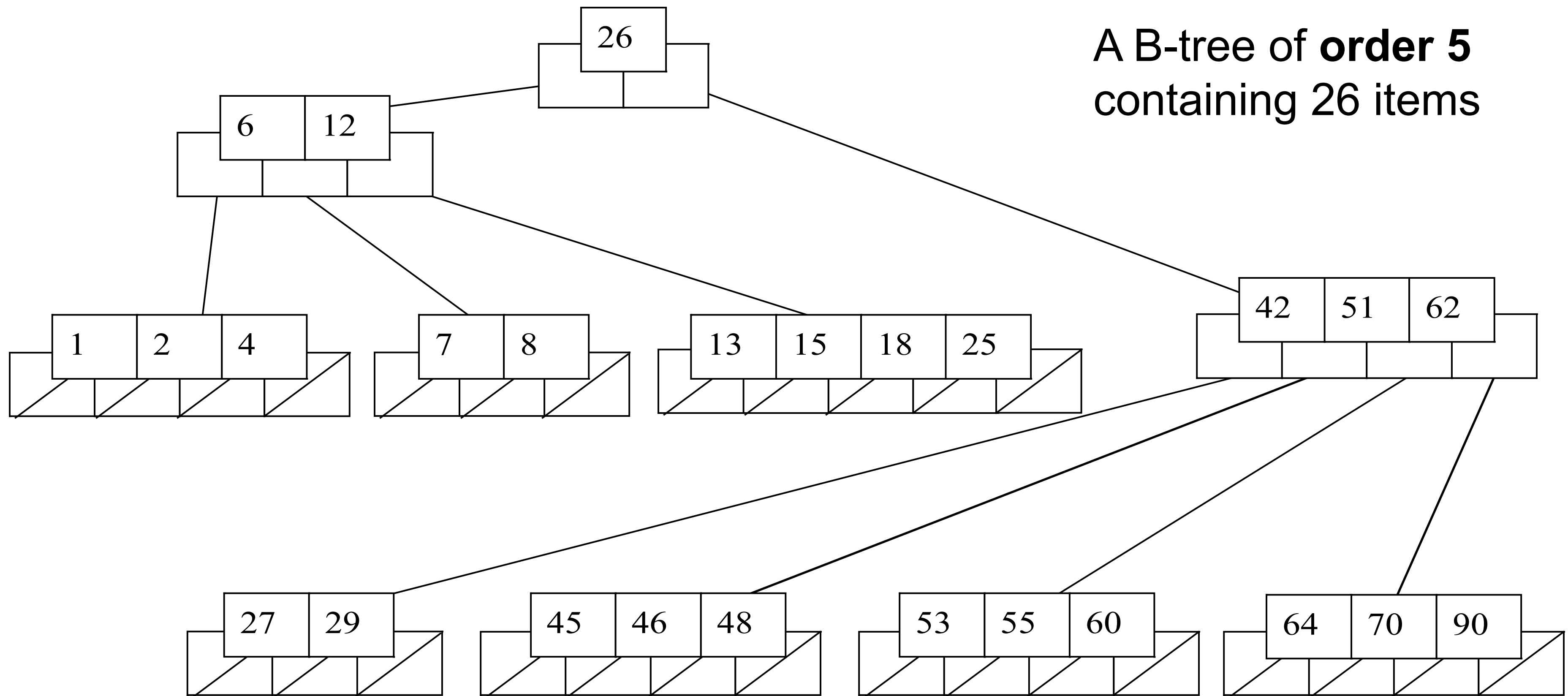
Motivation for B-Trees

- Assume that we use an AVL tree to store about 20 million records
- We end up with a **very** deep binary tree with lots of different disk accesses; $\log_2 20{,}000{,}000$ is about 24, so this takes about 0.2 seconds
- We know we can't improve on the log *n* lower bound on search for a binary tree
- **But, the solution is to use more branches and thus reduce the height of the tree!**
  - **As branching increases, depth decreases**

Definition of a B-tree

- A B-tree of order m is an m-way tree (i.e., a tree where each node may have up to m children) in which:
  1. the number of keys in each non-leaf node is one less than the number of its children and these keys partition the keys in the children in the fashion of a search tree
  2. all leaves are on the same level
  3. all non-leaf nodes except the root have at least $\lceil m / 2 \rceil$ children
  4. the root is either a leaf node, or it has from two to m children
  5. a leaf node contains no more than $m - 1$ keys
- The number m should always be odd

An example B-Tree



Note that all the leaves are at the same level

Inserting into a B-Tree

- Attempt to insert the new key into a leaf
- If this would result in that leaf becoming too big, split the leaf into two, promoting the middle key to the leaf's parent
- If this would result in the parent becoming too big, split the parent into two, promoting the middle key
- This strategy might have to be repeated all the way to the top
- If necessary, the root is split in two and the middle key is promoted to a new root, making the tree one level higher

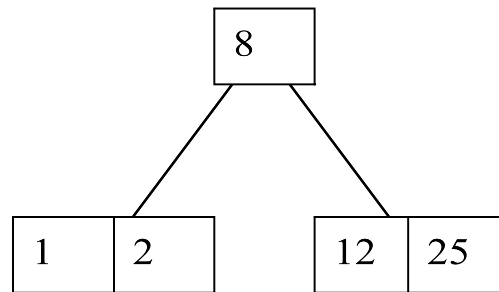
Constructing (Insertion) a B-tree

- Suppose we start with an empty B-tree and keys arrive in the following order:1 12 8 2 25 5 14 28 17 7 52 16 48 68 3 26 29 53 55 45
- We want to construct a B-tree of order 5
- The first four items go into the root:

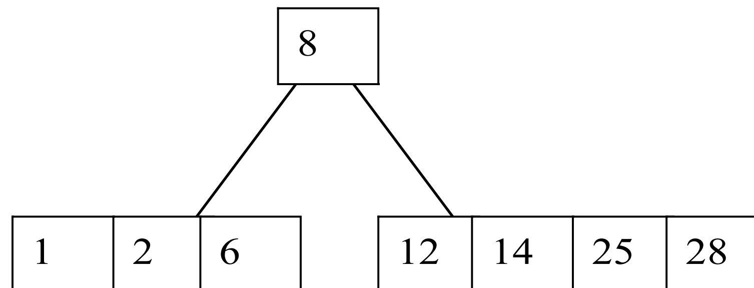
1	2	8	12

- To put the fifth item in the root would violate condition 5
- Therefore, when 25 arrives, pick the middle key to make a new root

Constructing a B-tree (contd.)



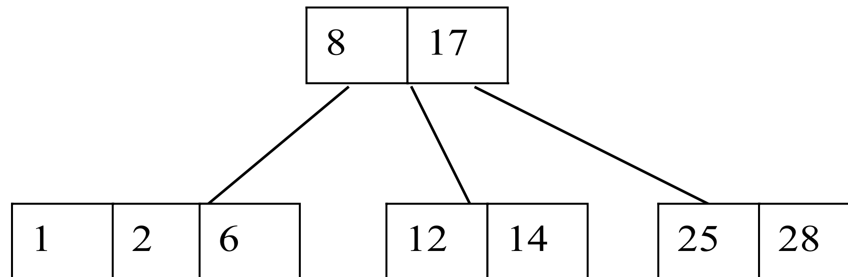
6, 14, 28 get added to the leaf nodes:



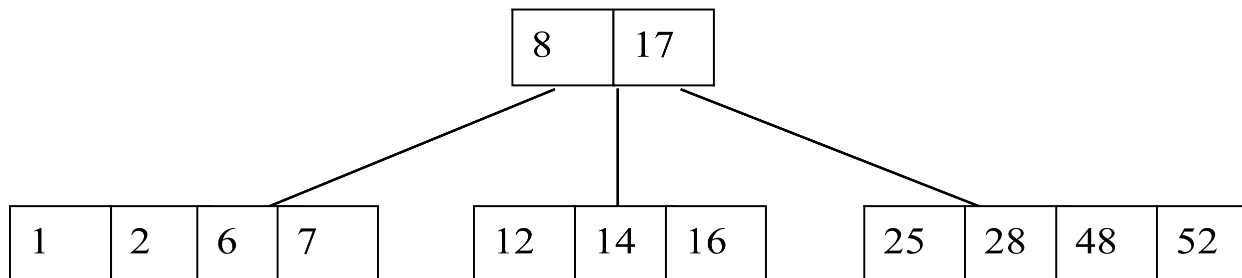
Add 17

Constructing a B-tree (contd.)

Adding 17 to the right leaf node would over-fill it, so we take the middle key, promote it (to the root) and split the leaf



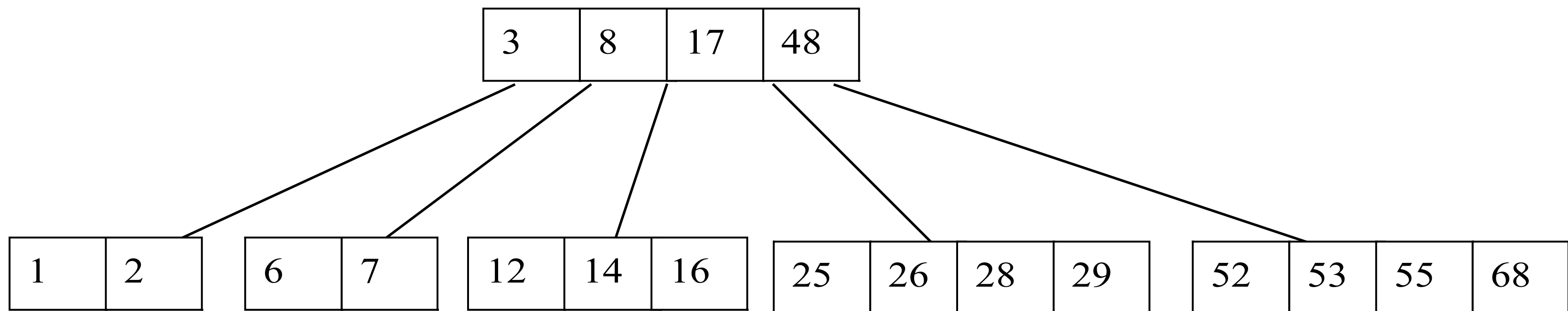
7, 52, 16, 48 get added to the leaf nodes



Add 68

Constructing a B-tree (contd.)

Adding 68 causes us to split the right most leaf, promoting 48 to the root, and adding 3 causes us to split the left most leaf, promoting 3 to the root; 26, 29, 53, 55 then go into the leaves

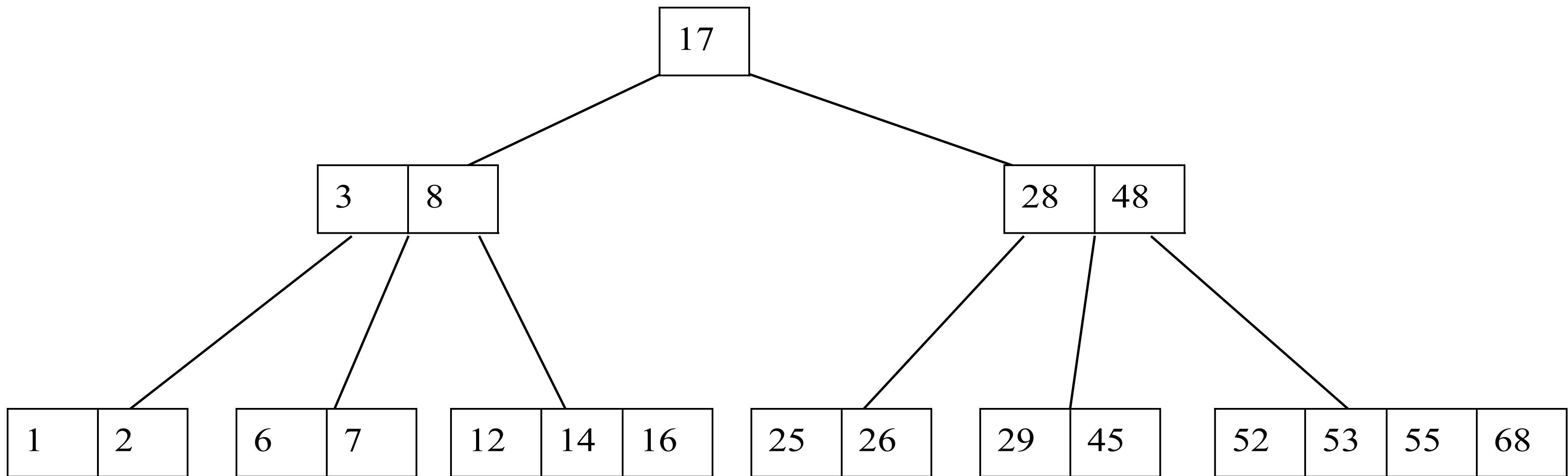


Adding 45 causes a split of

25	26	28	29

and promoting 28 to the root then causes the root to split

Constructing a B-tree (contd.)



Exercise in Inserting a B-Tree

- Insert the following keys to a 5-way B-tree:
- 3, 7, 9, 23, 45, 1, 5, 14, 25, 24, 13, 11, 8, 19, 4, 31, 35, 56
-

Removal from a B-tree

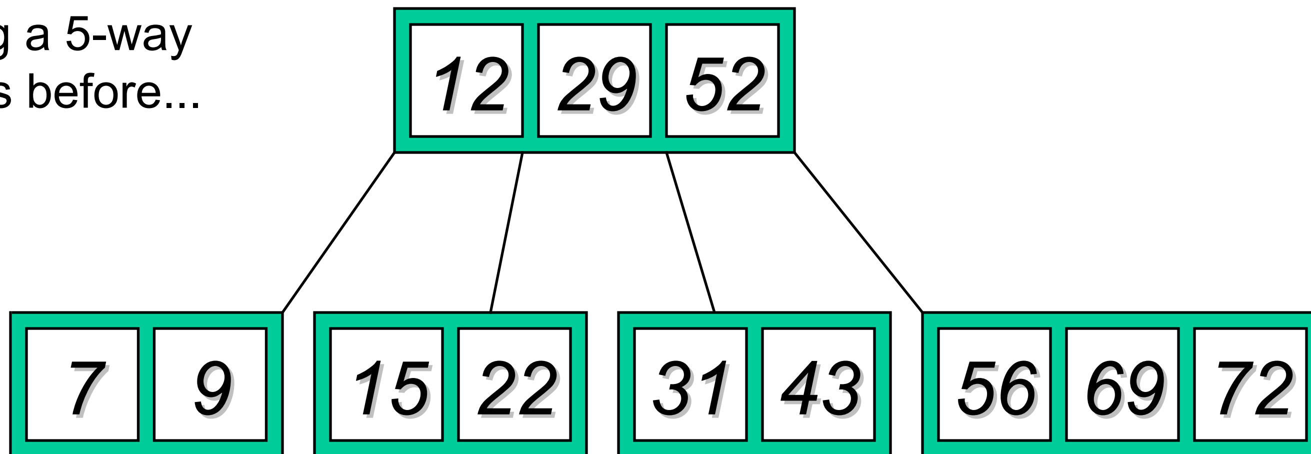
- During insertion, the key always goes *into* a *leaf*. For deletion we wish to remove *from* a leaf. There are three possible ways we can do this:
- 1 - If the key is already in a leaf node, and removing it doesn't cause that leaf node to have too few keys, then simply remove the key to be deleted.
- 2 - If the key is *not* in a leaf then it is guaranteed (by the nature of a B-tree) that **its predecessor or successor** will be in a **leaf -- in this case we can delete the key and promote the predecessor or successor key to the non-leaf deleted key's position.**

Removal from a B-tree (2)

- If (1) or (2) lead to a leaf node containing less than the minimum number of keys then we have to look at the siblings immediately adjacent to the leaf in question:
  - 3: if one of them has **more than the min. number of keys** then we can **promote one of its keys** to the parent and take the **parent key into our lacking leaf**
  - 4: if **neither** of them has **more than the min. number of keys** then the **lacking leaf** and one of its **neighbours** can be **combined** with their **shared parent** (the opposite of promoting a key) and the **new leaf** will have the correct number of keys; if this step leave the parent with too few keys then we repeat the process up to the root itself, if required

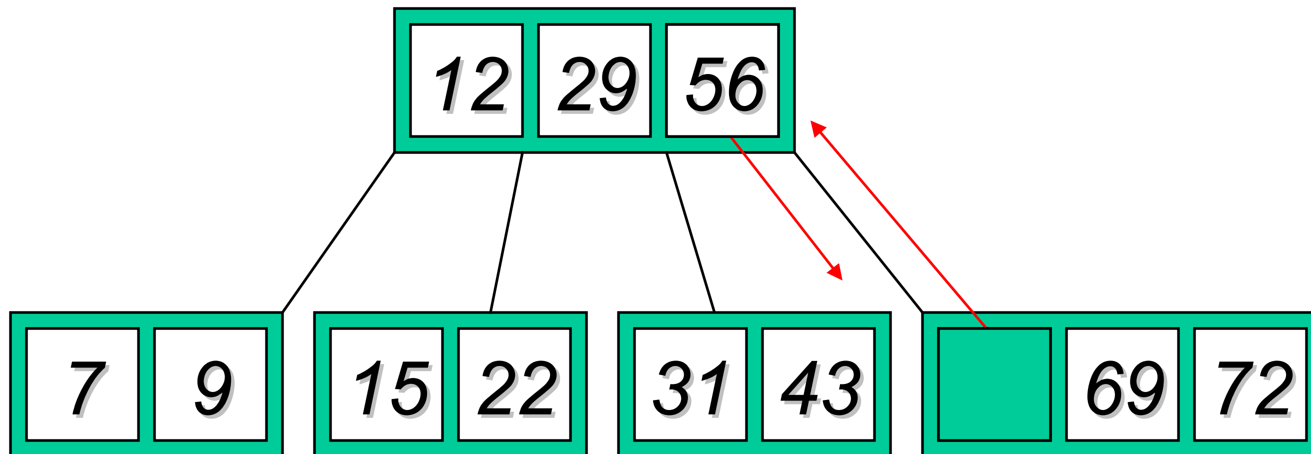
Type #1: Simple leaf deletion

Assuming a 5-way B-Tree, as before...

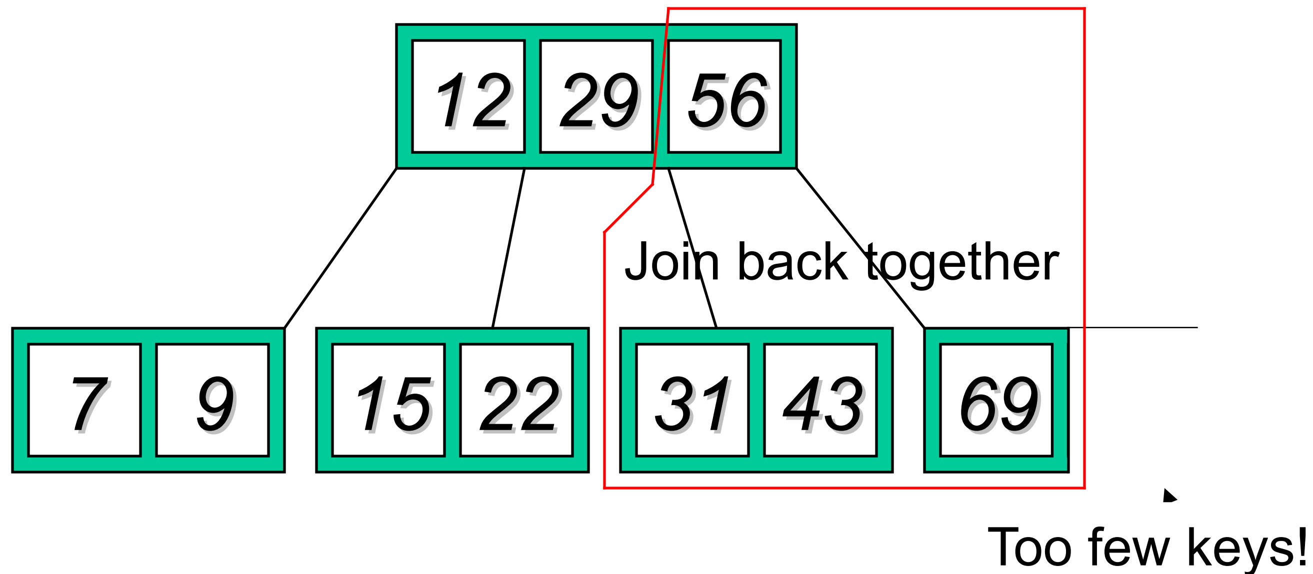


Delete 2: Since there are enough keys in the node, just delete it

Type #2: Simple non-leaf deletion



Type #4: Too few keys in node and its siblings



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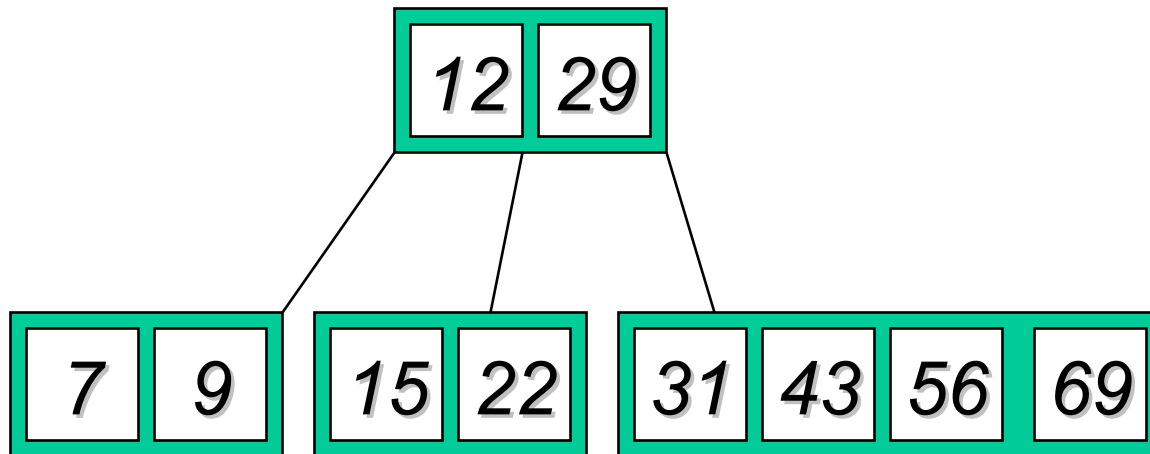
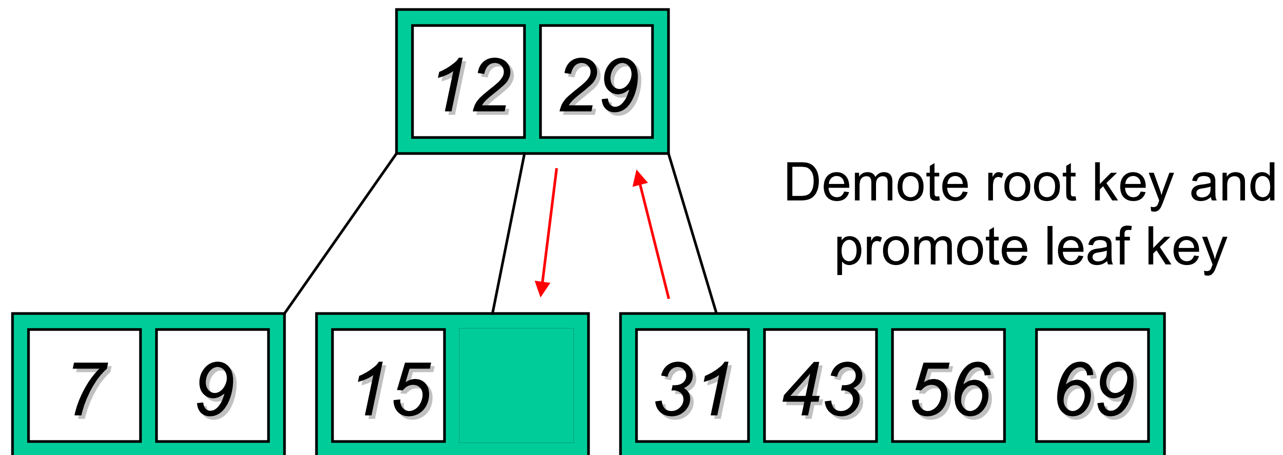


Type #3: Enough siblings



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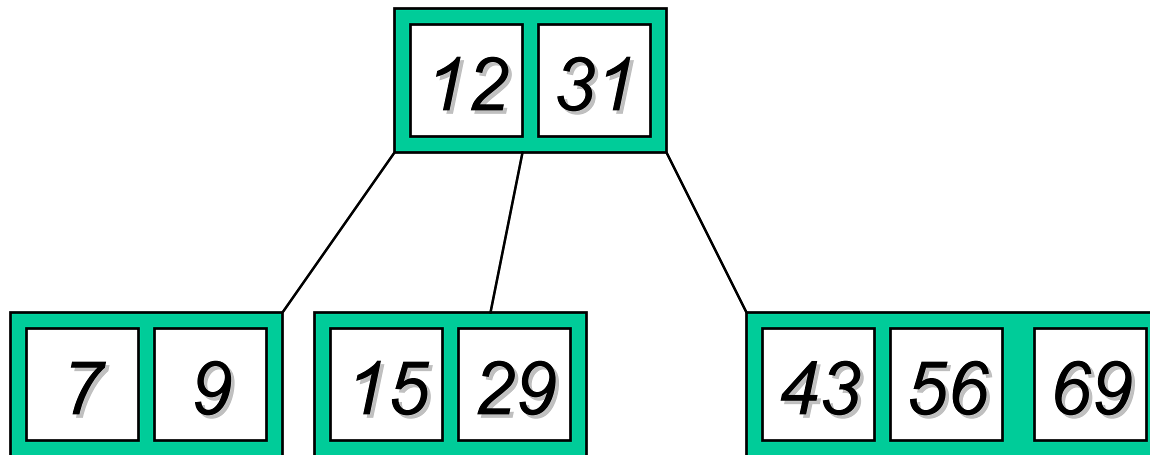

Exercise in Removal from a B-Tree

- Given 5-way B-tree created by these data (last exercise):
- 3, 7, 9, 23, 45, 1, 5, 14, 25, 24, 13, 11, 8, 19, 4, 31, 35, 56
- Add these further keys: 2, 6,12
- Delete these keys: 4, 5, 7, 3, 14
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Comparing Trees

- Binary trees
  - Can become *unbalanced* and *lose* their good time complexity (big O)
  - AVL trees are strict binary trees that *overcome the balance problem*
- Multi-way trees
  - B-Trees can be *m*-way, they can have any (odd) number of children
  - One B-Tree, the 2-3 (or 3-way) B-Tree, *approximates* a permanently balanced binary tree, exchanging the AVL tree's balancing operations for insertion and (more complex) deletion operations